

Practice Quiz KEY ch 2.1-2.4 Math 11

$$\textcircled{1} \left[\begin{array}{ccc|c} 6 & -6 & 30 & 24 \\ 3 & 2 & 5 & 7 \\ 5 & -2 & 15 & 5 \end{array} \right]$$

$$R1 \div 6 \rightarrow R1$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 5 & 4 \\ 3 & 2 & 5 & 7 \\ 5 & -2 & 15 & 5 \end{array} \right]$$

$$R2 - 3R1 \rightarrow R2$$

$$R3 - 5R1 \rightarrow R3$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 5 & 4 \\ 0 & 5 & -10 & -5 \\ 0 & 3 & -10 & -15 \end{array} \right]$$

$$R2 \div 5 \rightarrow R2$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 5 & 4 \\ 0 & 1 & -2 & -1 \\ 0 & 3 & -10 & -15 \end{array} \right]$$

$$R1 + R2 \rightarrow R1$$

$$R3 - 3R2 \rightarrow R3$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 3 & 3 \\ 0 & 1 & -2 & -1 \\ 0 & 0 & -4 & -12 \end{array} \right]$$

$$R3 \div (-4) \rightarrow R3$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 3 & 3 \\ 0 & 1 & -2 & -1 \\ 0 & 0 & 1 & 3 \end{array} \right]$$

$$R1 - 3R3 \rightarrow R1$$

$$R2 + 2R3 \rightarrow R2$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -6 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 3 \end{array} \right]$$

Solution $x = -6$ $y = 5$ $z = 3$

$$\textcircled{2} A = \begin{bmatrix} 1 & 3 & 0 \\ 2 & 1 & 1 \\ 3 & 4 & 2 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad B = \begin{bmatrix} 20 \\ 40 \\ 30 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 & 0 \\ 2 & 1 & 1 \\ 3 & 4 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 20 \\ 40 \\ 30 \end{bmatrix}$$

$$AX = B$$

$$A^{-1}AX = A^{-1}B$$

$$IX = A^{-1}B$$

$$X = A^{-1}B$$

$$A^{-1} = \begin{bmatrix} .4 & 1.2 & -.6 \\ .2 & -.4 & .2 \\ -1 & -1 & 1 \end{bmatrix} \text{ from calculator}$$

$$\text{so } X = A^{-1}B = \begin{bmatrix} .4 & 1.2 & -.6 \\ .2 & -.4 & .2 \\ -1 & -1 & 1 \end{bmatrix} \begin{bmatrix} 20 \\ 40 \\ 30 \end{bmatrix}$$

$$X = \begin{bmatrix} 38 \\ -6 \\ -30 \end{bmatrix} \quad \text{Solution}$$

$$x = 38$$

$$y = -6$$

$$z = -30$$

③ System 1: Reduced Row Echelon Form is $\left[\begin{array}{ccc|c} 1 & 0 & 9 & 12 \\ 0 & 1 & -7 & 15 \\ 0 & 0 & 0 & 0 \end{array} \right]$

The first equation is $x + 9z = 12$

$$\text{so } x = 12 - 9z$$

The second equation is $y - 7z = 15$

$$\text{so } y = 15 + 7z$$

Infinitely Many Solutions
(Dependent System)

Find some specific solutions by picking values of z and calculating x and y

$$\text{Ex: } z = 0, x = 12 - 9(0) = 12, y = 15 + 7(0) = 15$$

$$z = 1, x = 12 - 9(1) = 3, y = 15 + 7(1) = 22$$

$$z = 2, x = 12 - 9(2) = -6, y = 15 + 7(2) = 29$$

$$z = -1, x = 12 - 9(-1) = 21, y = 15 + 7(-1) = 8$$

$$z = -5, x = 12 - 9(-5) = 57, y = 15 + 7(-5) = -20$$

System 2: Reduced Row Echelon form is $\left[\begin{array}{ccc|c} 1 & 0 & 0 & 10 \\ 0 & 1 & 0 & -12 \\ 0 & 0 & 1 & -5 \end{array} \right]$

One solution

$$x = 10 \quad y = -12 \quad z = -5$$

System 3: Reduced Row Echelon form is $\left[\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array} \right]$

No Solution

4) a) $AB = \begin{bmatrix} a & b & c & d \\ 2 & -1 & 3 & 0 \end{bmatrix} \begin{bmatrix} 7 & 5 \\ -8 & 4 \end{bmatrix}$ not possible to find

The number of columns in A
does not equal
the number of rows in B

b) $BA = \begin{bmatrix} 7 & 5 \\ -8 & 4 \end{bmatrix} \begin{bmatrix} a & b & c & d \\ 2 & -1 & 3 & 0 \end{bmatrix}$

$$= \begin{bmatrix} 7a+5(2) & 7b+5(-1) & 7c+5(3) & 7d+5(0) \\ -8a+4(2) & -8b+4(-1) & -8c+4(3) & -8d+4(0) \end{bmatrix}$$

so $BA = \begin{bmatrix} 7a+10 & 7b-5 & 7c+15 & 7d \\ -8a+8 & -8b-4 & -8c+12 & -8d \end{bmatrix}$ dimensions 2×4

c) $B^2 = B \cdot B = \begin{bmatrix} 7 & 5 \\ -8 & 4 \end{bmatrix} \begin{bmatrix} 7 & 5 \\ -8 & 4 \end{bmatrix} = \begin{bmatrix} 7(7)+5(-8) & 7(5)+5(4) \\ -8(7)+4(-8) & -8(5)+4(4) \end{bmatrix}$

So $B^2 = \begin{bmatrix} 9 & 55 \\ -88 & -24 \end{bmatrix}$

d) $3B = 3 \begin{bmatrix} 7 & 5 \\ -8 & 4 \end{bmatrix} = \begin{bmatrix} 3(7) & 3(5) \\ 3(-8) & 3(4) \end{bmatrix} = \begin{bmatrix} 21 & 15 \\ -24 & 12 \end{bmatrix}$

e) Not possible to find $4A-2B$
because A and B have different dimensions
(A is 2×4 and B is 2×2)

f) $IA = A = \begin{bmatrix} a & b & c & d \\ 2 & -1 & 3 & 0 \end{bmatrix}$ $\begin{matrix} I & A \\ 2 \times 2 & 2 \times 4 \end{matrix}$ $\begin{matrix} I \text{ is identity} \\ \text{Matrix so} \\ IA = A \end{matrix}$
 $\xrightarrow{IA \text{ exists}}$

5) a) $\begin{matrix} D & E \\ (2 \times 3) & (3 \times 2) \\ \hline DE \text{ exists} \\ \downarrow \\ DE \text{ has} \\ \text{dimensions } 2 \times 2 \end{matrix}$

b) $\begin{matrix} E & D \\ (3 \times 2) & (2 \times 3) \\ \hline ED \text{ exists} \\ \downarrow \\ ED \text{ has} \\ \text{dimensions } 3 \times 3 \end{matrix}$

c) CD does not exist
Number of columns in C
does not equal
number of rows in D

d) $\begin{matrix} D & C \\ (2 \times 3) & (3 \times 3) \\ \hline DC \text{ exists} \\ \downarrow \\ DC \text{ has} \\ \text{dimensions } 2 \times 3 \end{matrix}$

e) EG does not exist
Number of columns in E
does not equal
number of rows in G

$$⑥ \quad a) \quad QP = \begin{bmatrix} 60 & 120 & 75 \\ 15 & 30 & 25 \end{bmatrix} \begin{bmatrix} 4 \\ .50 \\ 2 \end{bmatrix} = \begin{bmatrix} 60(4) + 120(.5) + 75(2) \\ 15(4) + 30(.5) + 25(2) \end{bmatrix}$$

so $QP = \begin{bmatrix} 450 \\ 125 \end{bmatrix}$ \$450 revenue from regular items
and \$125 revenue from gluten-free items

$$b) \quad T-Q = \begin{bmatrix} 55 & 130 & 65 \\ 12 & 36 & 20 \end{bmatrix} - \begin{bmatrix} 60 & 120 & 75 \\ 15 & 30 & 25 \end{bmatrix} = \begin{bmatrix} -5 & 10 & -10 \\ -3 & 6 & -5 \end{bmatrix}$$

Sales of regular bread decreased by 5 loaves
regular cookies increased by 10 cookies
regular muffins decreased by 10 muffins
gluten free bread decreased by 3 loaves
gluten free cookies increased by 6 cookies
gluten free muffins decreased by 5 muffins

$$c) \quad FTP = [1.20, .5] \begin{bmatrix} 55 & 130 & 65 \\ 12 & 36 & 20 \end{bmatrix} \begin{bmatrix} 4 \\ .50 \\ 2 \end{bmatrix}$$

$$FTP = \begin{matrix} \text{bread} & \text{cookies} & \text{muffins} \\ [66+6 & 156+18 & 78+10] \end{matrix} \begin{bmatrix} 4 \\ .5 \\ 2 \end{bmatrix}$$

$$= \begin{bmatrix} 72 & 174 & 88 \end{bmatrix} \begin{bmatrix} 4 \\ .5 \\ 2 \end{bmatrix} = [72(4) + 174(.5) + 88(2)] = [551]$$

Revenue from sales tomorrow is expected to be \$551 for these products, in total.